In the light of the information that students get much more involved during exercises than lectures (and actually get to understand things then) I have decided to "waste" yet another two hours on applying what we have already learned.

1 Integration

Integration is an important part of scientific and engineering work. In addition, it is well known that "most" integrations can not be carried out analytically; in other words, their results can not be represented as a finite sum of elementary functions. This is exactly where numerical integration comes into play.

How do we integrate numerically? One well-known method is the "trapezoidal rule". The formula is as follows:

$$\int_a^b f(x)dx = \sum_{k=1}^{N-1} f(a + k \cdot h) \cdot h + \frac{f(a) + f(b)}{2} \cdot h$$

where

$$h = \frac{b-a}{N}$$

2 Exercise: A Simple Integral

In this exercise, we will write a function that will calculate the following integral:

$$I = \int_a^b x^2 dx$$

The function prototype will be as follows:

```
float integrate(float a, float b, int N);
```

Obviously, `a` and `b` are the integration limits and `N` is the number of divisions.

After you have written the function, compare how the results approximate the true value of the integrals with increasing number of divisions of the interval.

3 Exercise: A Hard Integral

The normal distribution curve has the following form:

$$f(x) = \frac{1}{\sqrt{2\pi\sigma^2}} \cdot e^{\frac{(x-\overline{x})^2}{2\sigma^2}}$$

The integral of this function from minus to plus infinity is 1. However, integrals between finite values can not be calculated analytically. We will integrate the "normal" form of this function from zero to `x`:

$$I = \int_0^x \frac{1}{\sqrt{2\pi}} \cdot e^{y^2/2} dy$$

I approaches 0.5 as x approaches infinity.

Now, write a function that calculates this integral and has the following prototype:

```
float gaussian_integral(float x, int N);
```

Here, `x` is the upper limit of the integral, and `N` is the number of divisions.
Below are a few accurate values of this integral for verifying your work:

x	Integral
0.5	0.1914624613
1.0	0.3413447460
2.0	0.4772498680
3.0	0.4986501020
4.0	0.4999683288

For each value of `x`, find out how many divisions you need to use in order to get the answer correct to five decimal places. See how this number varies with `x`.