CSE 505 Problem Set 1

Date: Thursday, October 7th, 2004
Due date: Thursday, October 14th, 2004

- **Problem 1**
  Use the axioms of the algebra of events to prove the relations:
  a. $U' = \emptyset$
  b. $A + B = A + A'B + ABC$
  c. $A + U = U$

- **Problem 2**
  Find $P(A + (B' + C')')$ in each of the following cases:
  a. A, B, C are mutually exclusive events and $P(A) = 3/7$.
  b. $P(A) = 1/2$, $P(BC) = 1/3$, $P(AC) = 0$.
  c. $P(A'(B' + C')) = 0.65$.

- **Problem 3**
  Sonia and Norman are playing a friendly game of "Battleship" where each ship occupies only one square. Each player's grid is formed of $N \times N$ squares.

  Sonia places two ships on her grid, one after the other, in the following manner: for each ship, she makes an equally likely choice among all available squares in the grid. What is the probability that the two are adjacent? Note that adjacent squares are squares that share a common *edge*, sharing a common point is not adjacency. In other words, squares on the same diagonal are not considered to be adjacent.

- **Problem 4**
  Three cowboys, A, B and C, decide to have a shootout, where the last man standing wins. Their aiming abilities are different. A, which is a perfect shooter, definitely kills whom he shoots at. B, which is not as good, has a probability 1/2 to kill the person he shoots at. C, who is the worst shooter of the three, hits and kills with probability of only 1/3.

  They decide to take turns in shooting, where every shooter has to take one shot, which he must aim honsetly at one of the opponents (cowboy ethics!). Knowing each other's hitting probabilities, the cowboys decide that the shooting should go in the order C, B, A. When one cowboy dies, the other two continue taking turns.

  Assume that each cowboy decides whom to shoot at knowing the hit probabilities, and trying to maximize the probability of winning.

  a. Given these conditions, at whom should cowboy C shoot at in his first chance to maximize his chances of survival? What is the probability that he will survive in this case?

  b. What is the probability that B will survive (win the shootout)?

  a. What is the probability that A will survive (win the shootout)?