CSE 505 Problem Set 6

Date: Friday, November 21st, 2003
Due date: Friday, December 5th, 2003

- **Problem 1**

Determine the expected value, variance, and the transform of the PMF for the total number of trials from the start of a Bernoulli process (with parameter p) up to and including the nth success after the mth failure.

- **Problem 2**

Al performs an experiment comprising a series of independent trials. On each trial he simultaneously flips a set of three fair coins.

a. What is the probability that a trial with 3 heads occurs before a trial with at least two tails?

b. Given that Al just had a trial with 3 tails, what is the probability that both of the next two trials will also have this result?

c. Whenever all three coins land on the same side in any given trial, Al calls the trial a success.

(i) Find the PMF for K, the number of trials up to, but *not* including, the second success.

(ii) Find the expectation of L, the number of successes in the first 100 trials.

(iii) Find the expectation and variance of M, the number of tails that occur before the first success.

(iv) Find the PMF for N, the number of trials with 3 heads that occur in 100 trials.

(v) Find the conditional PMF for N, given $L = l$.

- **Problem 3**

A store opens at $t = 0$ and *potential* customers arrive in a Poisson manner at an average arrival rate of λ potential customers per hour. As long as the store is open, and independently of all other events, each particular potential customer becomes an *actual* customer with probability p. The store closes as soon as ten actual customers have arrived.

a. What is the probability that exactly three of the first five potential customers become actual customers?

b. What is the probability that the fifth potential customer to arrive becomes the third actual customer?

c. What is the PDF and expected value for L, the duration of the interval from store opening to store closing?

d. Given only that exactly three of the first five potential customers became actual customers, what is the conditional expected value of the total time the store is open?

e. Considering only customers arriving between $t = 0$ and the closing of the store, what is the probability that no two actual customers arrive within τ time units of each other?