CSE 505 Problem Set 2

Date: Thursday, October 2nd, 2003
Due date: Thursday, October 9th, 2003

• **Problem 1**

An urn initially contains one red and one blue ball. At each stage, a ball is randomly chosen (uniformly) and then replaced in the urn, along with another ball of the same color (that is, if the first selected ball is red, then that ball is placed back along with another red ball, making the urn contain two reds and one blue.) Let x denote the selection number of the first chosen ball that is blue. For example, if the first selected ball is red, and the second selected ball is blue, then $x = 2$. Assume that selections are independent.

a. Find $p_x(x_0)$, $x_0 \geq 1$.

b. Show that the event "the blue ball is eventually chosen" occurs with probability 1.

c. What can you say about the expected value of x? (That is, how long, on average, will it take to draw the blue ball?)

For the following three questions, let x and y be two discrete random variables.

• **Problem 2**

Prove Jensen's Inequality: If $f(x)$ is convex, $E[f(x)] \geq f(E[x])$.

Note: A function is said to be convex if for any two points x_1 and x_2 in its domain, and $0 \leq a_1, a_2 \leq 1$, and $a_1 + a_2 = 1$, we have:

$$f(a_1 x_1 + a_2 x_2) \leq a_1 f(x_1) + a_2 f(x_2)$$

• **Problem 3**

Prove the Cauchy-Schwarz inequality:

$$(E[xy])^2 \leq E[x^2]E[y^2]$$

• **Problem 4**

Given that x and y are positive, independent, and identically distributed, prove that:

$$E\left[\frac{x}{y}\right] \geq 1$$