CSE 505 Midterm Examination

Date: Tuesday, October 11^{th}, 2003
Duration: Two hours

• **Problem 1**

Player A and player B decide to play a game of dice, using a single, six-sided fair die. The game is to played as follows: Player A will roll the die first. If he rolls a six, he wins the game and the game ends. If he fails to roll a six, player B gets to roll the die. He wins if he rolls a six. If not, player A gets to roll again, and the game continues in the same way until one of the players rolls a six.

a. Find the probability that Player A wins.

b. Assuming each player wagers $1.00 on the game, what is the expected value of Player A's earnings?

c. For the game to be "fair", i.e., for the expected gain of both parties to be zero, how much should Player A wager for each game given Player B wagers $1.00?

• **Problem 2**

Let x_1 and x_2 be two independent experimental outcomes of a six-sided fair die. Define random variable $y = \max(x_1, x_2)$.

a. Find $p_y(y_0)$, the PMF of random variable y for all values of y_0.

b. Find $E(y)$ and σ_y^2.

• **Problem 3**

Two students, X and Y enter an examination. Their grades are represented by the random variables x and y, respectively. The joint PDF of their grades (which are between 0 and 1 and taken to be continuous) is given by:

$$f_{x,y}(x_0, y_0) = A \cdot (1 + x_0 y_0), \;\; 0 \le x, y \le 1$$

a. Find the value of the constant A.

b. Find the two marginal PDFs, $f_x(x_0)$ and $f_y(y_0)$.

c. Are the students honest? That is, are the two random variables x and y independent?

d. It is known that Y has scored at least 0.5 more than X. (In other words, it is known that $y - x > 0.5$.) What is the probability that Y has scored more than 0.75?

• **Problem 4**

Random variable x is described by the PDF:

$$f_x(x_0) = x_0 e^{-x_0}, \;\; x_0 > 0$$

a. Find $f_x^T(s)$, the s-transform of the PDF.

b. Calculate $E(x)$ and σ_x^2.

c. Calculate $E(e^{x/2})$.

d. Calculate $E(e^x)$