CSE 505 Final Examination

Date: Tuesday, January 6^{th}, 2004
Duration: Two hours

• **Problem 1**

Joe is performing an experiement by repeatedly rolling a pair of six-sided dice.

a. What is the probability that he rolls a total of seven before he rolls a total of eight?

b. He calls a roll a success if the numbers on the two dice are equal. Find the PMF for L, the number of rolls up to and including the sixth success.

c. Calculate the expectation value for M, the number of successes in 1000 rolls.

• **Problem 2**

On a computer network, there are three computers, A, B, and C, that can generate data packets. Each computer produces data packets in a Poisson manner, with rates λ_A, λ_B, and λ_C, respectively.

a. Assume we listen to packets on the network, disregarding the sources of packets. Calculate the PDF for L, the interarrival times of any two packets.

b. If we start listening to packets on the network at a random time, what is the probability that the first packet we catch is from computer A?

c. A packet from computer B is an ICMP packet with probability P. (Only computer B produces ICMP packets.) What is the PDF for interrarival times of ICMP packets on the network?

• **Problem 3**

a. A discrete-state discrete-transition Markov process has three states. The state transition probabilities at each step from state i to state j are given by the state transition matrix p_{ij}. It is known that $p_{ij} = p_{ji} > 0$ for all pairs (i, j). Find the limiting state probabilities P_i in terms of p_{ij}.

b. This time, consider a Markov process with N states, with the same property that $p_{ij} = p_{ji} > 0$. Find the limiting state probabilities P_i in terms of p_{ij}. (The result is very important for quantum statistical mechanics.)

• **Problem 4**

Two light bulbs, A and B, illuminate a room with no windows or other light source. Bulb A has an exponentially distributed lifetime, with an expectation of 1000 hours. In case bulb A fails, the attendant replaces the bulb. The time it takes to replace the bulb is (believe it or not!) also exponentially distributed, with an expected value of 50 hours. Bulb B is a special, long-lasting bulb that also has an exponentially distributed lifetime with an expected value of 5000 hours. However, if bulb B fails, a specialist must be called to replace it, and the replacement time for bulb B is exponentially distributed with an expected value of 500 hours.

a. Draw a Markov model for this process. Mark your states and transitions clearly.

b. What fraction of the time is the room in total darkness?

c. What fraction of the time is the room illuminated with two bulbs?

d. Which of the two bulbs illuminates the room on its own for longer stretches of time?